



ثالوية التكنولوجيا التطبيقية
Applied Technology High School

MATHEMATICS PROJECT

Term 3 - 11(Core subject)/2012



JOHN NAPIER.
OF MERCHISTON
INVENTOR OF THE LOGARITHMS

INVENTOR OF LOGARITHMS

Names:

Section:



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Marking Scheme

	Criteria				Points
	4	3	2	1	
Completeness of Tasks x 12.5	Tasks are totally completed and correct.	Tasks are partially completed, OR Partially wrong.	Tasks are partially completed, AND Partially wrong.	Tasks are Attempted	—
Integration of Technology x 5	Students used one mean of technology mastering this tool. The tool used helped the student and was useful to support his project	Student used a mean of technology but it was not that supportive to the topic.	-----	Student use of technology was primitive and way below the level of other IAT students	—
Presentation and Explanation Skills x 5	Student was able to explain the work he/she submitted confidently and fluently. Student reflected a deep understanding of his/her work; she was able to answer all of colleagues and instructor's questions.	Student was able to explain the work he/she submitted confidently and fluently. Student reflected an understanding of his/her work; he/she was able to answer most of colleagues and instructor's questions.	Student was able to explain the work he/she submitted. Student reflected a shallow understanding of his/her work; she was able to answer some of colleagues and instructor's questions.	Student was unable to explain the work he/she submitted. Student reflected no understanding of his/her work; he/she was unable to answer any of colleagues and instructor's questions.	—
Outstanding Tip x 2.5	Student had an outstanding addition in all aspects of his/ her project.	Student had an outstanding addition in some aspects of his/her project.	Student had an outstanding addition in very few aspects of his/her project.	Student had an outstanding addition in no aspects of his/her project.	—
This rubric is out of 100, percentage orientation. To make the mark out of 30 (Student's Mark/10*3)				Total---->	—



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Rules

- 1- Each group consists of three students.
- 2 - Students can present the project as a power point or any other software that supports the graphs and the mathematical calculations.
- 3- Students have to use a spreadsheet to perform part one (question 2), and it must be attached to the presentation.
- 4- The project should be organized and neat.
- 5- The project is to be discussed with each group to record the final mark.



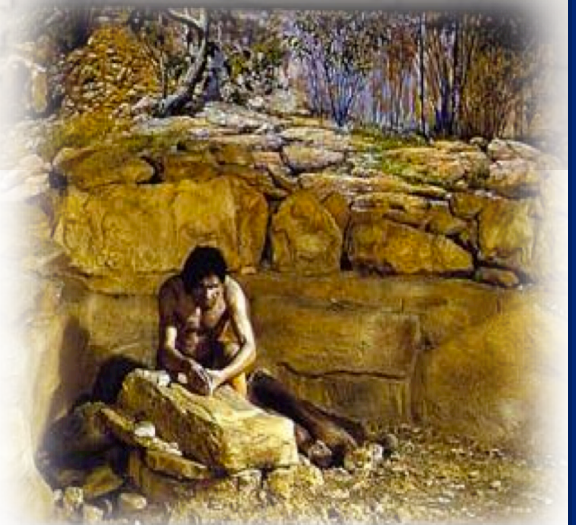
PART ONE

Radiocarbon Dating

In 1940 a group of boys walking in the woods near the village of Lascaux in France suddenly realized their dog was missing. They soon found him in a hole too deep for him to climb out. One of the boys was lowered into the hole to rescue the dog and stumbled upon one of the greatest archaeological discoveries of all time. What he discovered was a cave whose walls were covered with drawings of wild horses, cattle and a fierce-looking beast resembling a modern bull.

In addition, the cave contained the charcoal remains of a small fire, and from these remains scientists were able to determine that the cave was occupied 15,000 years ago.

All living things contain Carbon 14. When they die, the C-14 begins to decay. We can determine how long something has been dead by the amount of C-14 left.





Questions 1: By chemical analysis it has been determined that the amount of C-14 remaining in the samples of the Lascaux charcoal was 15% of the amount such trees would contain when living. The half-life of C-14 is approximately 5600 years. Write an equation to model this situation. *(HINT: Write the equation of the decay then find the value of the constant k. Then you can estimate the time using the results from the first step.)*

The half time of C-14 element is the time it takes for 1/2 the initial amount to decay.

Another model for the exponential decay is $y = ae^{kt}$ where k is a constant. This is the model preferred by scientists to measure the decay in the intensity of radioactive material over time.



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Equation:

Calculations to find k:

Calculations to find the time (t):

For your information..

Why do we use logarithms?

To measure earthquake intensity on the Richter scale. To measure the pH or acidity of a chemical solution, the pH is the negative logarithm of the concentration of free hydrogen ions. We also perceive brightness of light as the logarithm of the actual light energy, and star magnitudes are measured on a logarithmic scale.

Continuously compounded interest

Question 2: The average cost of a home in Tucson is roughly around \$225,000. Suppose you were planning to put down \$25,000 now and finance the rest on a 30-year mortgage at 7% compounded monthly. How much would your **monthly** payments be?

* Let's say that P_k represents the present monthly value k months ago.
* F is your monthly payment. (the value that we need to calculate)

Notes:

- 1) For a 30 -year mortgage, you'll be making 360 monthly payments.
- 2) At the end of the 360 months we want the *present value* (P) of all the monthly payments to add up to the amount you plan to finance, \$200,000

Step 1:

$$200,000 = P_1 + P_2 + P_3 + \cdots + P_{360} \quad \text{and} \quad P_k = F \left(1 + \frac{r}{n} \right)^{-k} \quad \text{so ,}$$

$$200,000 = F \left(1 + \frac{r}{n} \right)^{-1} + F \left(1 + \frac{r}{n} \right)^{-2} + \cdots + F \left(1 + \frac{r}{n} \right)^{-360}$$

And, accordingly with some algebra!



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$$\frac{200,000}{\left[\left(1 + \frac{r}{n}\right)^{-1} + \left(1 + \frac{r}{n}\right)^{-2} + \cdots + \left(1 + \frac{r}{n}\right)^{-360} \right]} = F$$

Notice that we need to figure out how to sum up the numbers in the table below. This can be done using **Excel**.

Step 2: Now you can start and find the monthly payment.

Since we're compounding monthly at 7%,
 $r = 0.07$ and $n = 12$

k	$(1+(r/n))^k$	P_k
1	0.9942005	1338.36183
2	0.98843463	1346.16894
3	0.9827022	1354.0216
4	0.97700301	1361.92006
5	0.97133688	1369.86459
6	0.96570361	1377.85547
.	?	?
.	?	?
.	?	?
.	?	?
.	?	?
.	?	?
.	?	?
.	?	?
358	?	?
359	?	?
360	?	?
Sum ?		



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Step 3:

a. Use the excel sheet to graph K versus P_k . What happens to your outstanding balance after each monthly payment is made?

Insert the graph given by the excel sheet here please.

b. Is this a graph of an exponential growth or decay?



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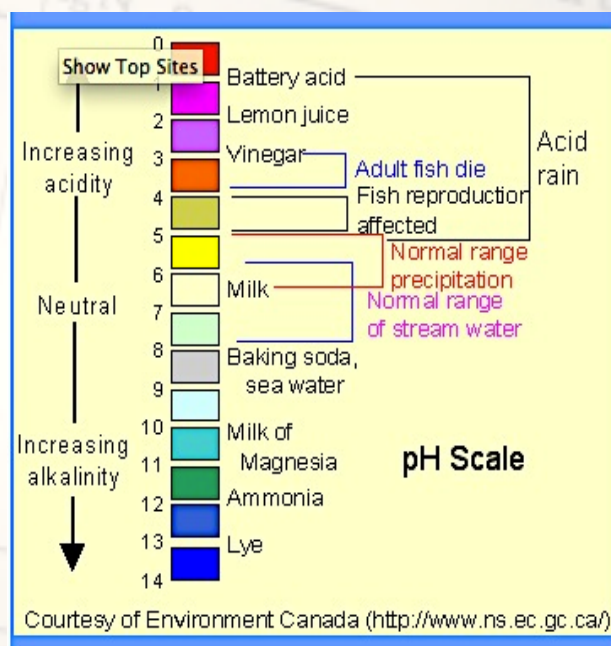
c. Justify your answer using the formula $P_k = F \left(1 + \frac{r}{n} \right)^{-k}$.

**** NOTE : Your excel sheet must be attached to the project**

PART TWO

Chemical acidity!

The pH scale measures how acidic or basic a substance is. The pH scale ranges from 0 to 14. A pH of 7 is neutral. **The pH scale is logarithmic** and as a result, each whole pH value below 7 is ten times more acidic than the next higher value.



Question 1: a. If an acid has an H^+ concentration of 0.0001 M, find the pH. Is it acidic or basic?

Step 1: _____

Step 2: _____

pH is defined as the negative logarithm of the hydrogen ion concentration. The equation is :
 $pH = -\log [H^+]$



b. A certain fruit has a pH of 2.2 and an antacid tablet has a pH of 10.1. How many times more is the concentration of hydrogen ion in fruit as compared to the antacid?

Loudness of sound

Question 2: "Loudness" is measured in decibels. The formula for the loudness of a sound is given by " $\text{dB} = 10 \log[I \div I_0]$ " where I_0 is the intensity of "threshold sound", or sound that can barely be perceived. Other sounds are defined in terms of how many times more intense they are than threshold sound.

a. Express the formula so that the expression is written as a single logarithm.



b. If a cat's purr is about 316 times as intense as threshold sound, then what is the decibel rating of the cat's purr?

b. Considering that prolonged exposure to sounds above 85 decibels can cause hearing damage or loss, and considering that a gunshot from a "22- rimfire" rifle has an intensity of about $I = (2.5 \times 10^{13}) I_0$, should you follow the rules and wear ear protection when relaxing at the rifle range?



Art of proofs!

Question 3: Many logarithmic expressions may be rewritten, either expanded or condensed. Expanding is breaking down a complicated expression into simpler components. Condensing is the reverse of this process.

Now, why is $\log_a x^n = n \log_a x$?



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Question 4: show that the sum $\log \frac{1}{2} + \log \frac{2}{3} + \log \frac{3}{4} + \log \frac{4}{5} \dots +$

$\log \frac{99}{100}$ equals to a constant. And find the constant.

End of the project
Good luck